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A Remark on Witten Effect for QCD Monopoles in Matrix Quantum Mechanics

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Abstract

In a recent work (hep-th/9805198) we argued that a certain matrix quantum mechanics may describe 't Hooft's monopoles which emerge in QCD when the theory is projected to its maximal Abelian subgroup. In this note we find further evidence which supports this interpretation. We study the theory with a non-zero theta-term. In this case, 't Hooft's QCD monopoles become dyons since they acquire electric charges due to the Witten effect. We calculate a potential between a dyon and an anti-dyon in the matrix quantum mechanics, and find that the attractive force between them grows as the theta angle increases.

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't Hooft has shown [1] that a new and rich structure emerges in QCD when certain unusual gauge fixing conditions are imposed on the theory. This class of unitary gauges projects QCD onto its maximal Abelian subgroup, *i.e.*, breaks $SU(N)$ down to $U(1)^{N-1}$. In addition to massless and massive gauge fields, which are present after this symmetry breaking, some point-like monopoles emerge in the theory [1]. These monopoles appear in a somewhat unusual way, as singularities of a chosen gauge fixing condition. Given the importance of these monopoles in various analytic and lattice studies of QCD (see [2] and references therein), it is desirable to have some gauge independent description of the dynamics of these objects. Recently, we addressed this issue in [3] by considering pure QCD on a spatial three-torus. The T-dual form of pure QCD on a spatial torus can be interpreted as a certain matrix quantum mechanics. In [3] some arguments were presented which lead us to conjecture that this quantum mechanics describes the dynamics of 't Hooft's monopoles. The aim of this note is to seek for further evidence in favor of this conjecture. Below we consider pure QCD with the theta term. It is known that 't Hooft's monopoles acquire electric charges due to the Witten effect [4] once non-zero theta angle is introduced [1]. If the identification of the excitations of the matrix quantum mechanics with 't Hooft's monopoles is correct, then the Witten effect should also be seen within the matrix model. In other words, if one calculates the interaction force between the point-like objects of the matrix model, then this force should depend on the theta angle. In fact, the attractive force between a monopole and an antimonopole should be greater then it is for a zero theta angle. In what follows we will show that this is indeed the case: An interacting monopole-antimonopole pair becomes a dyon-anti-dyon pair once the theta angle is switched on, and the attractive force between them grows as θ increases.

Consider four-dimensional pure $SU(N)$ QCD in the presence of the θ angle. The corresponding Lagrangian density reads:

$$\mathcal{L}_{\rm YM} = -\frac{1}{4g^2_{\rm YM}} G^a_{\mu\nu} G^{a\mu\nu} + \frac{\theta}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} = -\frac{1}{4} {\rm Re}\left(\tau \left[G^a_{\mu\nu} G^{a\mu\nu} + i G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}\right]\right) . \qquad (1)$$

Here $\widetilde{G}^{a\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G^a_{\lambda\rho}$, and $\tau = \tau_1 + i\tau_2 \equiv 1/g^2_{\rm YM} + i\theta/8\pi^2$. To avoid complications with Gribov copies, in the following we will be working in the $A_0 = 0$ gauge.

Let $\zeta \equiv 1/\Lambda_{\rm YM}$ be the effective correlation length of the theory, where $\Lambda_{\rm YM}$ is the dynamically generated QCD scale. Let us compactify the theory on a rectangular three-torus $T_L \equiv S^1 \times S^1 \times S^1$ with the radii of all three circles equal $L \gg \zeta$. This corresponds to the strong coupling regime of the theory (see discussions in [3]). We can rewrite pure QCD compactified on T_L as a matrix quantum mechanics compactified on a dual three-torus $T_R \equiv \widetilde{S}^1 \times \widetilde{S}^1 \times \widetilde{S}^1$ with the radii of all three circles equal $R \equiv \alpha'/L$, where the parameter α' is defined via the QCD scale $\Lambda_{\rm YM}$ as follows: $\alpha' \equiv \zeta^2 = 1/\Lambda^2_{\rm YM}$. In this T-dual formulation of the theory the dynamical variables are time-dependent matrices $\Phi_i(t)$, $i = 1, 2, 3$, transforming in the adjoint representation of (global) $SU(N)$. In addition to the color indices, for a given value of the index $i = 1, 2, 3$, the matrices Φ_i also carry indices corresponding to the winding modes. In general, Φ's give a matrix representation of a covariant derivative on a torus [5]. In the following we will suppress for simplicity these winding indices. The corresponding Lagrangian of the matrix quantum mechanics (with the appropriate normalization for Φ_i) is given by:

$$\mathcal{L} = \frac{1}{2g\sqrt{\alpha'}} \; \mathrm{Tr} \; \left(\dot{\Phi}_i^2 + \frac{1}{2(2\pi\alpha')^2} \left[\Phi_i, \Phi_j\right]^2 - \frac{i\lambda}{2\pi\alpha'} \epsilon_{ijk} \left[\Phi_i, \Phi_j\right] \dot{\Phi}_k \right) \; , \tag{2}$$

where $\dot{\Phi}_i$ denotes the time derivative of Φ_i, and the traces over color and winding indices (with appropriate normalizations) are implicit. This Lagrangian should be amended by a corresponding constraint equation (a counterpart of the Gauss's law) to describe pure QCD in a T-dual picture [3]. The new coupling constant g is defined as follows:

$$g \equiv (R/L)^{3/2} g_{\mathrm{YM}}^2 / 4\pi \; . \tag{3}$$

Also, the last term in (2) is due to the θ-term in (1), and the corresponding coupling λ is given by:

$$\lambda \equiv \tau_2/\tau_1 = \theta g_{\mathrm{YM}}^2 / 8\pi^2 \; . \tag{4}$$

For $\theta = 0$ (2) reduces to the usual bosonic matrix quantum mechanics Lagrangian [6–10,5,11].

Following [3] the Lagrangian (2) describes the dynamics of 't Hooft's QCD monopoles. Let us notice that in the limit $L \gg \zeta$, that is, $R \ll \zeta$, which we are interested in, the matrix quantum mechanics (2) is as complicated a theory as strongly coupled pure QCD, the reason being that it contains light *winding* modes (which map to the Kaluza-Klein modes in the T-dual QCD description) whose masses scale as $R/\alpha' = 1/L$ [3]. However, certain aspects of pure QCD in a large volume (which is a strongly coupled theory) might be more transparent in the matrix quantum mechanics approach. In particular, the monopole mass in the theory is given by $M = 1/2g\sqrt{\alpha'}$, and in the regime we are discussing $M \gg \Lambda_{\mathrm{YM}}$ [3]. Thus, it is reasonable to consider interactions between monopoles when they are moving very slowly (or, are almost at rest). At such a low energies the light winding modes are not yet excited in the model. Thus, we can neglect the contributions of these modes in the calculation. On the matrix model side interactions between monopoles (a monopole-antimonopole pair) are described by off-diagonal elements in Φ_i. Thus, following [3], consider the $U(2)$ case where we have two monopoles with opposite magnetic charges. Let us make the standard decomposition of the Φ field into it's classical and quantum parts:

$$\Phi_i = \Phi_i^{\mathrm{cl}} + \delta\Phi_i \; . \tag{5}$$

Here we choose the classical solution as follows:

$$\Phi_1^{\mathrm{cl}} \;=\; \frac{1}{2} \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix} \;, \quad \Phi_2^{\mathrm{cl}} = 0 \;, \quad \Phi_3^{\mathrm{cl}} = 0 \; . \tag{6}$$

That is, the two monopoles are at a distance r apart from each other in one of the spatial directions. In order to find an effective potential between them one performs integration with respect to off-diagonal fluctuations. In the absence of the θ-angle the effective potential between the monopoles is given by (see [3] and references therein):

$$V_{\mathrm{eff}}^{(\theta=0)} \propto \frac{r}{\alpha'} \; . \tag{7}$$

In fact, the easiest way to deduce the effective potential in the presence of the θ-angle is to rewrite the Lagrangian (2) as follows:

$$\mathcal{L} = \frac{1}{2g\sqrt{\alpha'}} \text{ Tr } \left(\left(\dot{\Phi}_i - \frac{i\lambda}{2(2\pi\alpha')} \epsilon_{ijk} \left[\Phi_i, \Phi_j \right] \right)^2 + \frac{1+\lambda^2}{2(2\pi\alpha')^2} \left[\Phi_i, \Phi_j \right]^2 \right) . \tag{8}$$

Note that the difference between the Lagrangians with $\lambda = 0$ and $\lambda \neq 0$ is in the redefinition of the conjugate momentum and rescaling $\alpha' \to \alpha'/\sqrt{1+\lambda^2}$ in the term containing the commutator $[\Phi_i, \Phi_j]^2$ (which is responsible for interactions between monopoles)[1]. Performing explicitly integration of the off-diagonal excitations in (8), and calculating the corresponding functional determinant, one finds the effective potential in the presence of the θ-angle

$$V_{\text{eff}}^{(\theta\neq 0)} \;=\; \sqrt{1+\lambda^2}\; V_{\text{eff}}^{(\theta=0)} \; . \tag{9}$$

We see that, as in the case without the θ-angle, there is a linearly rising potential between a monopole and an anti-monopole. Thus, there is a string stretched between them, and the string tension T_s has a non-trivial θ-dependence:

$$T_s \propto \sqrt{1+\lambda^2}\; \Lambda^2_{\text{YM}} = \sqrt{1 + (\theta g^2_{\text{YM}}/8\pi^2)^2}\; \Lambda^2_{\text{YM}} \; . \tag{10}$$

As a result, the attraction force between the pair increases when the theta angle is switched on. This corresponds to the fact that monopoles acquire electric charges and become dyons. This is consistent with the fact that we expect Witten's effect [4] to take place for magnetic monopoles - a monopole with a magnetic charge h becomes a dyon with the electric charge

$$e = \Big(\frac{\theta g^2_{\text{YM}}}{16\pi^2}\Big)\; h \tag{11}$$

in the presence of the θ angle [4,1]. Thus, the fact that the interaction force derived from the above matrix quantum mechanics depends non-trivially on the θ-angle gives additional evidence that 't Hooft's QCD monopoles might indeed be described by the former. Dyons in this case can (very roughly) be thought of as complicated bound states of a monopole and off-diagonal gluons.

Let us also point out that the string tension T_s is invariant under the S-duality transformation $\tau \to 1/\tau$. On the other hand, at first it might appear strange that it is not invariant under the shift $\theta \to \theta + 2\pi$. This is, however, expected as the electric charge e given by (11) is not invariant under such shifts either; the charge (11) gets shifted by a fundamental unit of the "electric charge" (which in our notations is $g^2_{\text{YM}} h/8\pi$), *i.e.*, $e \to e + g^2_{\text{YM}} h/8\pi$. This can be interpreted as follows: 't Hooft's dyons at θ can be viewed as a bound state of the corresponding dyon at $\theta - 2\pi$ and a gluon [1].

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[1]This is, however, not true for the Hamiltonian of the theory which will contain a term linear in theta along with the quadratic term arising in front of the commutator $[\Phi_i, \Phi_j]^2$.

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